Design of Two-Model IMM Estimators for Tracking Maneuvering Targets

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***Abstract*—The Interacting Multiple Model (IMM) estimator is well accepted as the best algorithm for tracking maneuvering targets, when the computational cost is considered. The IMM estimator includes a model-conditioned estimator for each kinematic model and the switching between modes or models is assumed to be a finite state Markov chain. The two-model configuration of the IMM estimator, the most commonly used version, typically includes either two nearly constant velocity (NCV) motion models or one NCV model and one nearly constant acceleration (NCA) model. In this paper, the design of these two configurations of the IMM estimator is considered. In this case, design refers to the selection of the motion models (*i.e.*, NCV or NCA) and the corresponding process noise variances. The design methods are first considered for single coordinate tracking with measurements of position, and simulation results are given to illustrate the effectiveness of the design methods. Then, the design methods are applied to radar tracking, and simulation results are given to demonstrate the effectiveness of the design methods.**

***Index Terms*—Target tracking, Kalman filter, radar systems, estimation, multiple model estimator**

I. Introduction

The Interacting Multiple Model (IMM) estimator is well accepted as the best algorithm for tracking maneuvering targets, when the computational cost is considered [1][3][10]. The IMM estimator includes a model-conditioned estimator for each kinematic model and the switching between modes (*i.e.*, models) is assumed to be a finite state Markov chain. The two-model configuration of the IMM estimator is the most commonly used version. The two-model version typically includes either two nearly constant velocity (NCV) motion models or one NCV model and one nearly constant acceleration (NCA) model. Typically, an NCV model with a small process noise variance is used for the nonmaneuver mode of motion, and either an NCV or NCA model is used for the maneuver mode of motion. The selection of either NCV or NCA for maneuvers has historically been an ad hoc process as no systematic process for model order selection existed prior to [9]. Similarly, the selection of the process noise variance for the maneuver model has historically been an ad hoc process that involves the use of extensive Monte Carlo simulations. Some design guidelines for process noise selection are given in [1].

In this paper, design methods are developed for the two-model configurations of the IMM estimator that utilize the NCV and NCA models. In both configurations, an NCV model with a small process noise variance is used for the nonmaneuver mode. The results of [9] are used to provide insight into the selection of an NCV or NCA model for the maneuver mode. When the NCV model is selected for the maneuver mode, the design methods of [8] are used for selecting the process noise variance. When the NCA model is selected for the maneuver mode, the design methods of [6] are used for selecting the process noise variance. In both cases, the process noise variance that minimizes the maximum mean squared error (MinMaxMSE) is chosen for the maneuver model. The design methods are first developed for single coordinate tracking with measurements of position, and simulation results are given to illustrate the effectiveness of the design methods. Then, the design methods are applied to radar tracking [5] [8], and simulation results are given to demonstrate their effectiveness.

This paper is organized as follows. In Section II, background on tracking maneuvering targets with the IMM estimator is given. That includes the system state model for Markovian switching systems, the NCV model with discrete white noise acceleration (DWNA) and design of the NCV Kalman filter, the NCA model with Discrete Wiener Process Acceleration (DWPA) and design of the NCA Kalman filter, and selection between the NCV and NCA models. Design methods for the IMM Estimator with two NCV models (IMM-CVCV) are developed in Section III, and results of Monte Carlo simulations are given to illustrate the effectiveness of the design methods. The design the IMM estimator for tracking with linear measurements of position is addressed first, and then tracking with radar measurements is considered next. The design of the IMM Estimator with NCV and NCA models (IMM-CVCA) is addressed in Section IV and simulation results are given to illustrate the effectiveness of the design methods. Concluding remarks are given in Section V.

II. Background

Some background on tracking maneuvering targets with the IMM estimator is given in this section.

A. Markovian Switching System for Modeling Maneuvering Target Tracking

Let X_k denote the kinematic state vector of the target at time t_k, and it typically contains the position, velocity, and possibly acceleration of the target as well as other variables used to model a time-varying acceleration. The kinematic model commonly assumed for a maneuvering target in track [1] is given by

$$X_{k+1} = F_k(\theta_{k+1})X_k + G_k(\theta_{k+1})v_k(\theta_{k+1}) \tag{1}$$

where $F_k(\theta_{k+1})$ defines the linear constraint on the target kinematics between times t_k and t_{k+1}, and $v_k(\theta_{k+1}) \sim N(0, Q_k(\theta_{k+1}))$ is the white process noise errors that account for uncertainty in the linear dynamics. The θ_{k+1} is a pointer to one of N models that describes the motion of the target from time t_k to t_{k+1}. The θ_{k+1} is treated as a finite state Markov chain with probability p_{ij} of switching between Model i at time t_k and Model j at time t_{k+1}.

The model for linear measurements is a linear model of the target state in Cartesian coordinates and given by

$$Z_k = H_k(\theta_k)X_k + w_k \tag{2}$$

where Z_k is typically a measurement of the position of the target and $w_k \sim N(0, R_k)$ is the white noise observation errors. Both w_k and $v_k(\theta_{k+1})$ are assumed to be independent white Gaussian error processes. For this paper, the linear measurements are scalar measurements of position, and the covariance R_k is the variance of the position measurements σ_w^2.

The model for radar measurements is a nonlinear model of the target state in Cartesian coordinates and given by

$$Z_k = h_k(X_k, \theta_k) + w_k \tag{3}$$

where Z_k is the typical radar measurement of range and angles for the target and $w_k \sim N(0, R_k)$ is the white noise observation errors. Both w_k and $v_k(\theta_{k+1})$ are assumed to be independent white Gaussian error processes. For this paper, the radar measures range, azimuth, and elevation, and the covariance R_k includes variances of the measurements in the range σ_r^2, azimuth σ_a^2,, and elevation σ_e^2.

B. IMM Estimator

The IMM estimator is well accepted as the best approach for estimating the state of a Markovian switching system defined by (1) and (2), when the computational cost is considered [1]. The IMM estimator is often employed to filter the kinematic measurements for estimating the target state [1]. The IMM estimator is a predictor-corrector algorithm with a time-update or prediction followed by measurement-update as found in the typical Kalman filter. In addition to the model-conditioned step, the IMM estimator also includes a state mixing step, a model likelihood evaluation step, mode probability update step, and an output blending combination step. Thus, the IMM estimator is often summarized in five steps.

The IMM algorithm consists of a Kalman filter[1] for each model, a model probability evaluator, an estimate mixer at the input of the filters, and an estimate combiner at the output of the filters. The multiple models interact through the mixing to track a target maneuvering through an arbitrary trajectory. Let $X_{k|k}$ denote the state estimate for time t_k based on all models and measurements through t_k, and let $P_{k|k}$ denote the corresponding state covariance. Let $X_{k|k}^j$ denote the state estimate based on Model j, and $P_{k|k}^j$ denote the corresponding state covariance. Let Λ_k be the vector of model likelihoods, while μ_k is the vector of model probabilities when all likelihoods have been considered. When using the IMM estimator for tracking maneuvering targets, Model 1, M_k^1, is typically an NCV model with a very small process noise variance, while Model 2, M_k^2, is a maneuver model that may be an NCV model with a larger process noise variance or an NCA model. With the assumption that the switching is governed by an underlying Markov chain with known model switching probabilities (part of the design parameters), the mixer uses the model probabilities and model switching probabilities to compute a mixed estimate for each model. At the beginning of a filtering cycle, each filter uses a mixed estimate and latest measurement to compute a new state estimate conditioned on the model within the filter and a likelihood that the model within the filter is correct. The likelihoods, prior mode probabilities, and mode-switching probabilities are used to compute new mode probabilities. The overall state estimate is then computed using the new state estimates and their model probabilities. The IMM estimator algorithm for tracking with N models is given in [1].

C. NCV Kalman Filter With DWNA

For the NCV model with DWNA [1], X_k consists of position and velocity. Let the motion of the target be defined in a single coordinate and the measurements be the positions of the target (*i.e.,* a linear function of the state). For the NCV model with DWNA in a single coordinate, the state and measurement equations of (1) and (2) are defined in [1]. Let T denote the time interval between measurements. For a scalar coordinate, let $R_k = \sigma_w^2$ be the variance of the measurement errors in m^2, and $Q_k = \sigma_{NCV}^2$ be the variance of the "acceleration" errors in $\mathrm{m}^2/\mathrm{s}^4$.

The deterministic tracking index, Γ_{D}, was introduced in [4][8] as

$$\Gamma_{\mathrm{D}} = \frac{T^2 A_{max}}{\sigma_w} \tag{4}$$

and used to develop a relationship between the anticipated maximum acceleration of the target, A_{max}, and the process noise variance, σ_{NCV}^2, that achieves MinMaxMSE in position. By selecting the optimal process noise variance σ_{NCV}^2 for the NCV filter, the maximum mean squared error (MaxMSE) is minimized throughout the maneuver. The optimal process

[1]If the maneuver model or the measurement model is nonlinear, then an Extended Kalman Filter (EKF) is used.

noise variance is given by [8] in terms of the maximum acceleration of the target according to

$$\sigma_{NCV} = \kappa_1 A_{max} \tag{5}$$

where

$$\kappa_1 = 1.70(.66)^{\bar{\Gamma}_D}(1.02)^{\bar{\Gamma}_D^2}, \ 0.001 \leq \Gamma_D \leq 10 \tag{6}$$

and $\bar{\Gamma}_D = \log(\Gamma_D)$ with log being the base 10 logarithm. For the design of the NCV Kalman filter, the A_{max} used in Γ_D is the maximum increment in target acceleration that is sustained or persists for a few measurements.

D. NCA Kalman Filter with DWPA

In order to relax the assumption of zero-mean white noise acceleration errors of the NCV Kalman filter with DWNA, the NCA motion model is introduced to reduce the modeling mismatch of the acceleration errors as zero-mean, white noise [1]. For the NCA Kalman filter with DWPA in a single coordinate [1], the state and measurement equations of (1) and (2) are defined in [1]. Let $R_k = \sigma_w^2$ with σ_w^2 being the variance of position measurement errors in m^2, $E[v_k^2] = \sigma_{NCA}^2$ be the variance of the DWPA increment, and T is the time interval between the measurements at times t_k and t_{k+1}.

When designing NCA Kalman filters with DWPA so that the error in position achieves MinMaxMSE for a specified Γ_D, the optimal process noise variance is found using the anticipated maximum increment in acceleration of the target according to

$$\sigma_{NCA} = \kappa_3 A_{max} \tag{7}$$

The values of κ_3 that correspond to MinMaxMSE in position is given by [6] as a function of Γ_D

$$\kappa_3 = 0.59(1.5)^{\bar{\Gamma}_D}(0.94)^{\bar{\Gamma}_D^2}(0.98)^{\bar{\Gamma}_D^3} \tag{8}$$

where $\bar{\Gamma}_D = \log(\Gamma_D)$.

E. NCV Model Versus NCA Model

Using the higher order NCA filter leads to higher variances for the position and velocity estimates than the NCV filter given identical measurement statistics. When the target is not maneuvering, these higher variances are suboptimal. Hence, when should acceleration be included in the target state model? A prerequisite to this question is the significance or meaningfulness of the acceleration estimates that are achievable. In other words, if the measurement quality and rate and number of measurements during a maneuver are insufficient to estimate acceleration that is better than assuming zero, then acceleration should not be estimated using an NCA model. In order to answer this prerequisite question, the acceleration estimation problem can be formulated as a parameter estimation problem using a quadratic polynomial [1]. Since parameter estimation assumes a "perfect" motion model, the variance of the acceleration estimate will be the minimum value achievable.

Let λ denote the number of standard deviations of the acceleration estimate contained in the interval of $[-A_{max} \ A_{max}]$. Figure 1 shows the minimum N number of measurements for estimation of a meaningful acceleration for $\lambda = 1$ and 3 versus Γ_D [9]. For no prior data, $\lambda = 3$ in Figure 1 gives a good sense of the number of measurements needed to get a meaningful estimate of acceleration. For a track filter with prior data, $\lambda = 1$ gives a better sense of the number of measurements needed to get a meaningful estimate of acceleration. Considering $\Gamma_D = 1$ in Figure 1 suggests that 4 measurements are required for a meaningful estimate of acceleration for $\lambda = 1$ and 6 measurements for $\lambda = 3$. If one doubles the data rate (*i.e.,* T becomes $T/2$), Γ_D becomes $0.25\Gamma_D = 0.25$ and 7 measurements are required for a meaningful acceleration estimate for $\lambda = 1$ and 10 measurements for $\lambda = 3$. In these cases, doubling the data rate gives a meaningful estimate of acceleration in 0.875 of the original time for $\lambda = 1$ and 0.833 of the original time for $\lambda = 3$. Thus, doubling the data rate does not halve the time required to get a meaningful acceleration estimate in these cases. Thus, when selecting between an NCV and NCA models for the maneuver mode of the IMM estimator, Γ_D and the number of measurements anticipated during a maneuver should be used with Figure 1 to make the choice. If maneuvers are anticipated to persist more than the minimum number of measurements, the NCA model should be used for the maneuver mode. Otherwise, the NCV model should be used for the maneuver mode.

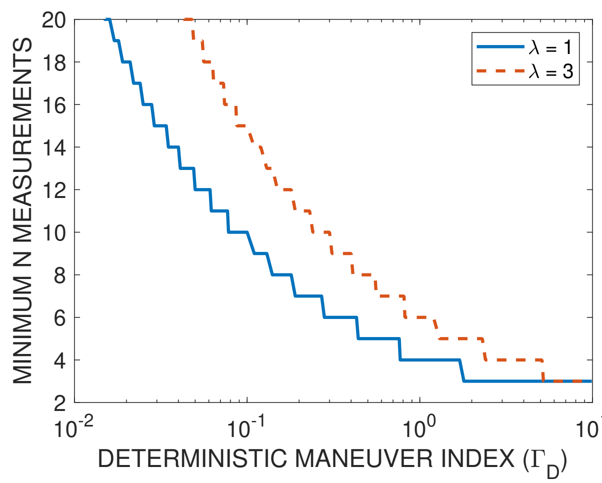

Fig. 1. Minimum Number of Measurements N for estimation of a meaningful acceleration defined for $\lambda = 1$ and 3 versus Γ_D

III. Design of IMM Estimator with Two NCV Models

Two NCV models are commonly used in the IMM Estimator. The NCV model is used for the maneuver mode when the quality and rate of the measurements and the duration of maneuvers do not allow for estimation of a meaningful acceleration. The design of the IMM estimator with two NCV models is considered in this section.

A. 1D Motion with Linear Measurements

For design of the IMM-CVCV, process noise variance for Model 1 is set to $\sigma_{NCV,1} = 1.0$ m/s^2 for regimes of flight in which the target is not maneuvering. The process noise variance for Model 2 for tracking through target maneuvers is selected as specified in [4] [8] for a target with a maximum acceleration of A_{max} according to

$$\sigma_{NCV,2} = \kappa_1 A_{max} \tag{9}$$

where κ_1 is given by (6). For this research, the Markov switching probabilities for the IMM estimator are specified by

$$p_{11} = 0.9 + 0.1e^{-\frac{T}{2}} \tag{10}$$
$$p_{22} = 0.8 + 0.2e^{-\frac{T}{2}} \tag{11}$$

with $p_{12} = 1 - p_{11}$ and $p_{21} = 1 - p_{22}$. These expressions for p_{11} and p_{22} are derived from past experience and nothing is systematic or automatic or optimal in their selection.

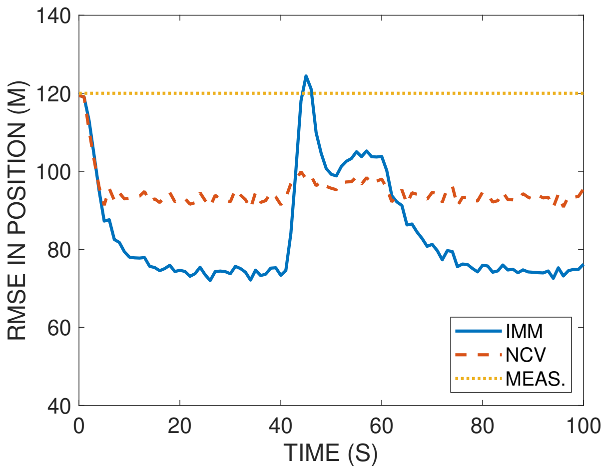

Fig. 2. RMSEs in position for IMM-CVCV and NCV Kalman filter for Case 1 with $T = 1.0$ s and $\sigma_w = 120$ m and designed for MinMaxMSE, $\sigma_{NCV} = \sigma_{NCV,2} = 84$ m/s^2.

Monte Carlo simulations were conducted to illustrate the effectiveness the design methods. In this study, the first target travels with a constant velocity for 100 m/s except for a constant acceleration maneuver of $A_{max} = 40$ m/s^2 from 40 to 60 s. The sensor measures the position of the maneuvering target. Three cases are studied. For Case 1, $T = 1.0$ s and $\sigma_w = 120$ m giving $\Gamma_D = 0.33$, $\kappa_1 = 2.1$, and $\sigma_{NCV} = \sigma_{NCV,2} = 84$ m/s^2. Figure 2 provides the results of Monte Carlo simulations for the IMM Estimator and NCV Kalman filter for Case 1. Note that the IMM-CVCV designed as described here gives similar results to the NCV Kalman filter during maneuvers and significantly smaller errors when the target is not maneuvering. The IMM-CVCV does experience a peak error near that of the measurements at the beginning of the maneuver. This peak depends on the measurement quality relative the maneuver and the measurement rate.

For Case 2, $T = 0.25$ s and $\sigma_w = 120$ m giving $\Gamma_D = 0.02$, $\kappa_1 = 3.6$, and $\sigma_{NCV} = \sigma_{NCV,2} = 146$ m/s^2. Figure 3 provides the results of Monte Carlo simulations for Case 2. The IMM-CVCV designed as described here provides slightly larger errors compared to that of the NCV Kalman filter during maneuvers and smaller errors when the target is not maneuvering. For this case with measurements at a rate four times that in Case 1, the peak error at the beginning of the maneuver is not present in the performance of the IMM-CVCV. This is due to the higher measurement rate.

For Case 3, $T = 1$ s and $\sigma_w = 40$ m giving $\Gamma_D = 1.0$, $\kappa_1 = 1.7$ and $\sigma_{NCV} = \sigma_{NCV,2} = 68$ m/s^2. Figure 4 provides the results for Case 3. Again, the IMM-CVCV designed as described here provides similar results to the NCV Kalman filter during maneuvers and significantly smaller errors when the target is not maneuvering. Again, for measurements at 1 Hz, the IMM-CVCV experiences a peak error larger than the measurement errors for one measurement. The smaller measurement errors in this case make it more difficult to avoid the peak error of the IMM-CVCV exceeding the measurement error.

In Cases 1-3, the NCV Kalman filter clearly achieves MinMaxMSE as designed. The IMM-CVCV estimator designed as proposed here provides significantly better tracking when the target is not maneuvering and approximately achieves MinMaxMSE during maneuvers. The selection of the mode switching probabilities of (10) and (11) as a function of sample period has not been optimized. It is likely that one will find that p_{22} was chosen two small for these cases.

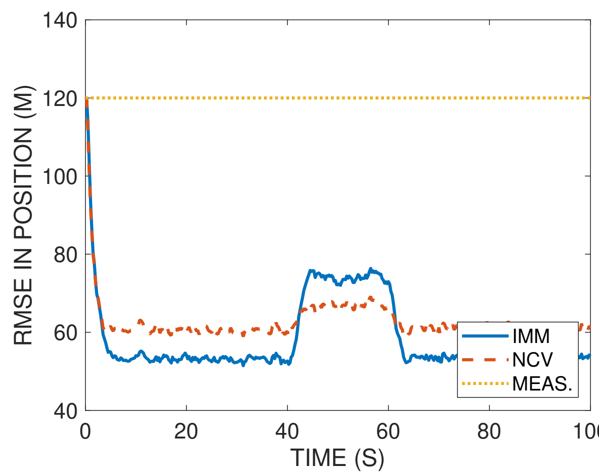

Fig. 3. RMSEs in position for IMM-CVCV and NCV Kalman filter for Case 2 with $T = 0.25$ s and $\sigma_w = 120$ m and designed for MinMaxMSE, $\sigma_{NCV} = \sigma_{NCV,2} = 146$ m/s^2.

B. Radar Tracking

Radar tracking involves processing measurements in range and angles with a target state defined in Cartesian coordinates. When tracking a target with a radar, the position error uncertainty or covariance ellipsoid tends to be oriented with the axes in the range and cross-range directions. Thus, for radar tracking, the design methods are applied in range and cross-range. The variance of the measurement errors in range are for the most part independent of range, while the standard deviation of the errors in cross-range varies linearly with the range of the target. The design of an IMM-CVCV for a radar

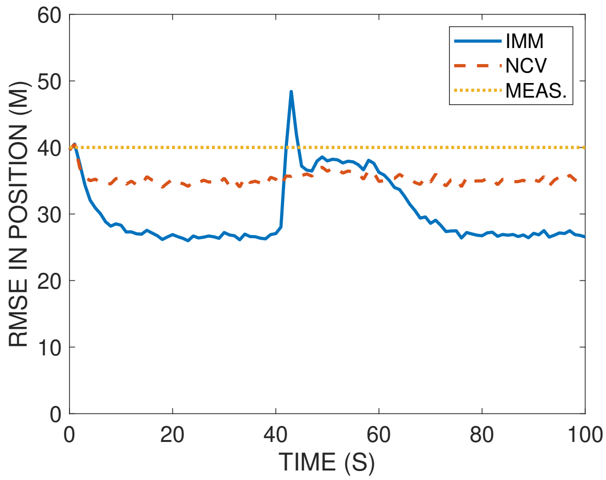

Fig. 4. RMSEs in position for IMM-CVCV and NCV Kalman filter for Case 3 with $T = 1.0$ s and $\sigma_w = 120$ m and designed for MinMaxMSE, $\sigma_{NCV} = \sigma_{NCV,2} = 146$ m/s^2.

tracking problem is addressed along the lines of the NCV Kalman filter with DWNA in [5] [8].

The IMM-CVCV and NCV Kalman filter track the target state in a Cartesian East-North-Up (ENU) coordinate system centered at the radar and processes measurements of range, azimuth, and elevation. The azimuth is measured counterclockwise from the East x-axis. The y-axis is North and the z-axis is Up. The radar measures target position in terms of range, azimuth, and elevation. The design criterion is MinMaxMSE in position using (9). Targets that turn with a maximum acceleration of 40 m/s^2 for less than 12 s are expected.

For this radar tracking problem, the cross-range errors will dominate the tracking errors. Thus, the deterministic tracking index is defined in cross-range as a function of range (r) by

$$\Gamma_D(r) = \frac{T^2 A_{max}}{\sqrt{3}\ r\ \max\{\sigma_a, \sigma_e\}}, \quad 0.001 \le \Gamma_D(r) \le 10 \quad (12)$$

where $A_{max} = 40$ m/s^2 in this problem and the $\sqrt{3}$ is included to reflect the maneuver acceleration projected onto three coordinates. If $\Gamma_D(r)$ falls outside the limits, it is fixed at the closest value within the bounds. Then, for MinMaxMSE, the process noise variance for the NCV Kalman filter is computed for radar tracking according to [8] as

$$\sigma_{NCV}(r) = \kappa_1(r)\frac{A_{max}}{\sqrt{3}} \quad (13)$$

$$\kappa_1(r) = 1.70(0.66)^{\bar{\Gamma}_D(r)}(1.02)^{\bar{\Gamma}^2_D(r)} \quad (14)$$
$$\text{for} \quad 0.001 \le \Gamma_D(r) \le 10$$

For design of the IMM estimator with two NCV models, process noise variance for Model 1 is set to $\sigma^2_{NCV,1} = 1.0$ m/s^2 for regimes of flight in which the target is not maneuvering. The process noise variance for Model 2 for tracking through target maneuvers is selected as specified in [8] for a target with a maximum acceleration of A_{max} according to

$$\sigma_{NCV,2}(r) = \kappa_1(r)A_{max} \quad (15)$$

where $\kappa_1(r)$ is given by (14).

The target starts near a range of 130 km on the x-axis with an altitude of 0 and travels toward the radar at a speed of 230 m/s. The target maintains a constant speed throughout the scenario. It turns left in the horizontal plane with 30 m/s^2 from 92 to 106 s and turns right on the horizontal plane with 30 m/s^2 from 132 to 146 s. It then turns left and up in a tilted plane near 45 degrees with 40 m/s^2 from 240 to 250 s and turns right and down in a titled plane near 45 degrees with 40 m/s^2 from 274 to 284 s. The target continues with a constant velocity until 350 s.

Three cases, Cases 4-6 are considered in the study of IMM-CVCV design for radar tracking. For Case 4, $T = 1$ s and $\sigma_a = \sigma_e = 1$ mrad, and Figure 5 gives the results for the Monte Carlo simulations that were conducted to illustrate the performance of the IMM-CVCV and NCV Kalman filter designed according (13) and (15), respectively. Note that the IMM-CVCV designed as described here gives similar results to the NCV Kalman filter during maneuvers and significantly smaller errors when the target is not maneuvering. The IMM-CVCV does not experience a peak error near that of the measurements at the beginning of the maneuver as in Case 1, because the accurate range measurements provide rapid maneuver response. For Case 5, $T = 0.25$ s, $\sigma_r = 5$ m, and $\sigma_a = \sigma_e = 1$ mrad, and Figure 6 gives the results for the IMM-CVCV and NCV Kalman filter. Note that the IMM-CVCV designed as described here gives slightly larger errors than the NCV Kalman filter during maneuvers and significantly smaller errors when the target is not maneuvering. The larger errors during maneuvers could be due to the approximated model switching probabilities of (10) and (11). For Case 6, $T = 1$ s and $\sigma_a = \sigma_e = 2.5$ mrad, and Figure 7 gives the results to illustrate tracking performance for the IMM-CVCV and NCV Kalman filter. Note that the IMM-CVCV designed as described here gives similar results to the NCV Kalman filter during maneuvers and significantly smaller errors when the target is not maneuvering. In Cases 4-6, the NCV Kalman filter clearly achieves MinMaxMSE as designed. The IMM-CVCV estimator designed as proposed here provides significantly better tracking when the target is not maneuvering and approximately achieves MinMaxMSE during maneuvers. The selection of the mode switching probabilities of (10) and (11) as a function of sample period has not been optimized.

IV. Design of IMM Estimator with NCV and NCA Models

It is common practice to use an NCV model and an NCA model in the IMM Estimator. The NCA model should only be used for the maneuver mode when quality and rate of the measurements and duration of maneuvers in number of measurements warrant the estimation of acceleration. Figure 1 gives insight into the conditions when a meaningful acceleration can be estimated. This section addresses the design of the IMM estimator with NCV and NCA models.

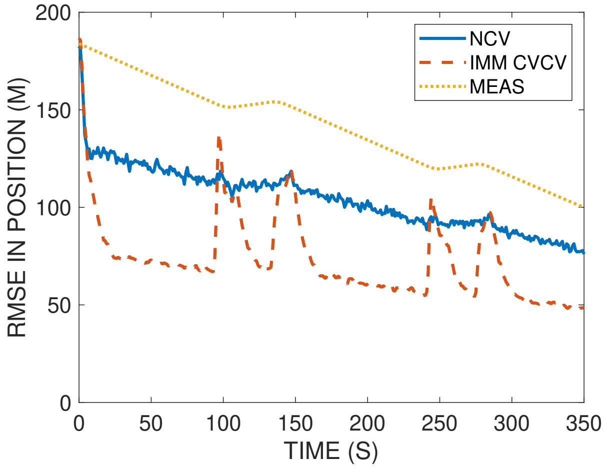

Fig. 5. RMSEs in position for IMM-CVCV and NCV Kalman filter radar tracking for Case 4 with $T = 1.0$ s, $\sigma_r = 5$ m, and $\sigma_a = \sigma_e = 1$ mrad and designed for MinMaxMSE via (13) and (15).

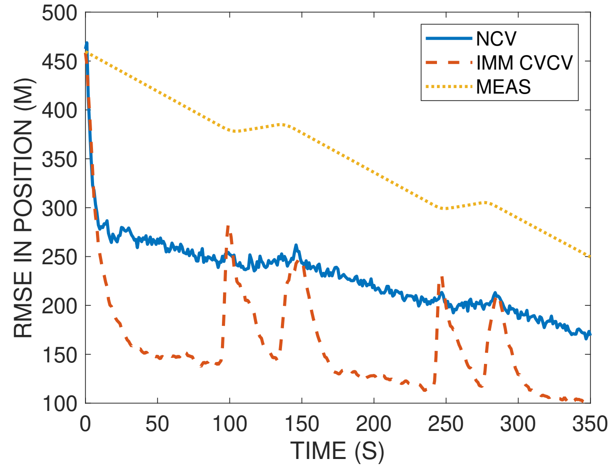

Fig. 7. RMSEs in position for IMM-CVCV and NCV Kalman filter for radar tracking for Case 6 with $T = 1.0$ s, $\sigma_r = 5$ m, and $\sigma_a = \sigma_e = 2.5$ mrad and designed for MinMaxMSE via (13) and (15).

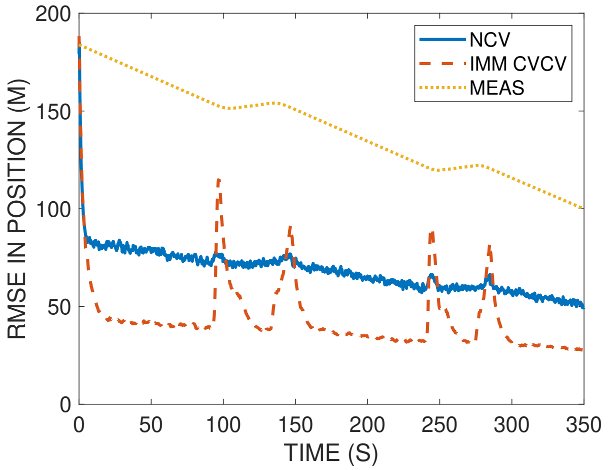

Fig. 6. RMSEs in position for IMM-CVCV and NCV Kalman filter radar tracking for Case 5 with $T = 0.25$ s, $\sigma_r = 5$ m, and $\sigma_a = \sigma_e = 1$ mrad and designed for MinMaxMSE via (13) and (15).

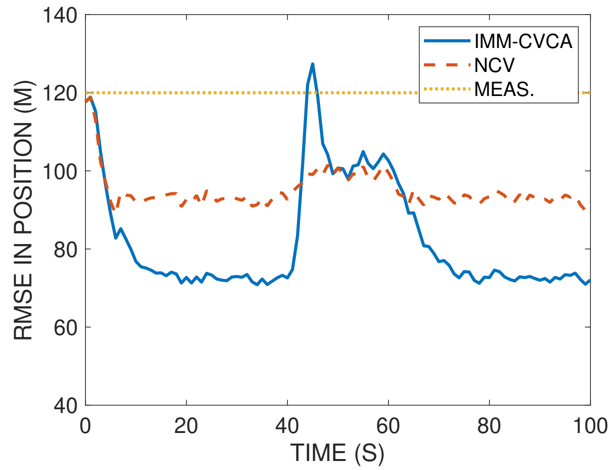

Fig. 8. RMSEs in position for IMM-CVCA and NCV Kalman filter for Case 7 with $T = 1.0$ s and $\sigma_w = 120$ m and designed for MinMaxMSE, $\sigma_{NCV} = 84$ m/s^2 and $\sigma_{NCA,2} = 18.9$ m/s^2.

A. 1D Motion with Linear Measurements

For design of the IMM estimator with NCV and NCA models, process noise variance for Model 1 is set to $\sigma^2_{NCV,1} = 1.0$ m/s^2 for regimes of flight in which the target is not maneuvering. The process noise variance for Model 2 for tracking through target maneuvers is selected as specified in (7) for a target with a maximum acceleration of A_{max} according to

$$\sigma_{NCA,2} = \kappa_3 A_{max} \tag{16}$$

where κ_3 is given by (8). For this research, the Markov switching probabilities for the IMM estimator are defined by (10) and (11).

Sensor parameters and target from Cases 1-3 are used for Cases 7-9. For Case 7, the $T = 1.0$ s and $\sigma_w = 120$ m giving $\Gamma_D = 0.33$, $\kappa_1 = 2.1$, and $\sigma_{NCV} = 84$ m/s^2. For the NCA model of the IMM Estimator, $\kappa_3 = 0.47$, and $\sigma_{NCA,2} = 18.9$ m/s^2. Figure 8 provides the results of Monte Carlo simulations for the IMM-CVCA and NCV Kalman filter for Case 7. Note that the IMM-CVCA designed as described here gives similar results to the NCV Kalman filter during maneuvers and significantly smaller errors when the target is not maneuvering. The IMM-CVCA does experience a peak error near that of the measurements at the beginning of the maneuver. This peak depends on the measurement quality relative the maneuver and the measurement rate. For Case 8, the $T = 0.25$ s and $\sigma_w = 120$ m giving $\Gamma_D = 0.02$, $\kappa_1 = 3.6$, and $\sigma_{NCV} = 146$ m/s^2. For the NCA model of the IMM Estimator, $\kappa_3 = 0.27$, and $\sigma_{NCA,2} = 10.8$ m/s^2. Figure 9 provides the results of Monte Carlo simulations for Case 8. Again, the IMM-CVCA designed as described here provides similar results to the NCV Kalman filter during maneuvers and smaller errors when the target is not maneuvering. For this case with measurements at a rate four times that in Case 7, the peak error at the beginning of the maneuver is not present in the performance of the IMM-CVCA. This is due to the higher measurement rate. For Case 9, $T = 1$ s and $\sigma_w = 40$ m giving $\Gamma_D = 1.0$, $\kappa_1 = 1.7$ and $\sigma_{NCV} = 68$ m/s^2. For the NCA model of the IMM Estimator, $\kappa_3 = 0.6$, and $\sigma_{NCA,2} = 24$ m/s^2. Figure 10 provides the results for Case 9. Again, the IMM-CVCA designed as described here provides similar results to the NCV Kalman filter during maneuvers and significantly smaller errors when the target is not maneuvering. Again, for measurements at 1 Hz, the IMM-CVCA experiences a peak error notably larger than the measurement errors for one measurement. The smaller measurement errors in this case make it more difficult to avoid the peak error of the IMM-CVCA exceeding the measurement error. In Cases 7-9, the NCV Kalman filter clearly achieves MinMaxMSE as designed. The IMM-CVCA estimator designed as proposed here provides significantly better tracking when the target is not maneuvering and approximately achieves MinMaxMSE during maneuvers.

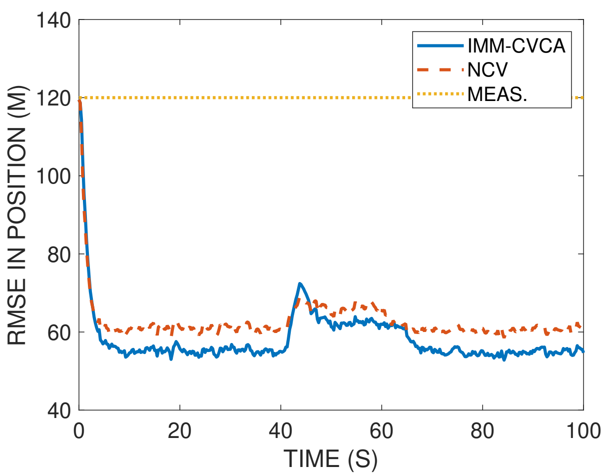

Fig. 9. RMSEs in position for IMM-CVCA and and NCV Kalman filter for Case 8 with $T = 0.25$ s and $\sigma_w = 120$ m and designed for MinMaxMSE, $\sigma_{NCV} = 146$ and $\sigma_{NCA,2} = 10.8$ m/s^2.

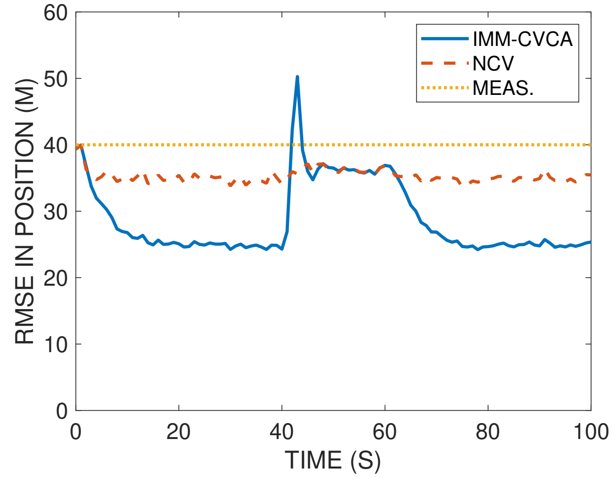

Fig. 10. RMSEs in position for IMM-CVCA and and NCV Kalman filter for Case 9 with $T = 1.0$ s and $\sigma_w = 40$ m and designed for MinMaxMSE, $\sigma_{NCV} = 68$ and $\sigma_{NCA,2} = 24$ m/s^2.

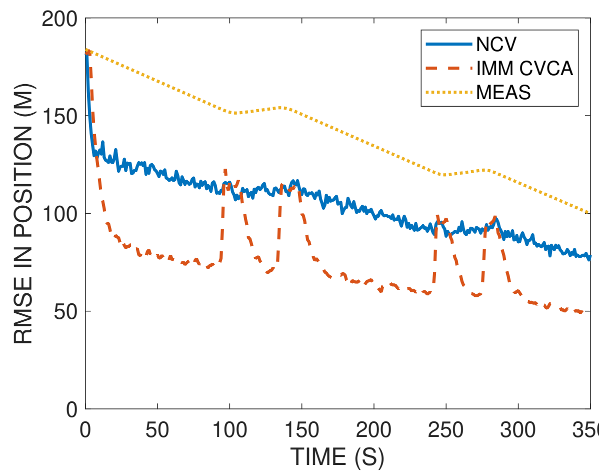

Fig. 11. RMSEs in position for IMM-CVCA and NCV Kalman filter radar tracking for Case 10 with $T = 1.0$ s, $\sigma_r = 5$ m, and $\sigma_a = \sigma_e = 1$ mrad and designed for MinMaxMSE via (13) and (17).

B. Radar Tracking

For design of the IMM-CVCA for the radar tracking problem, the cross-range errors will dominate the tracking errors. For design of the IMM-CVCA, process noise variance for Model 1 is set to $\sigma^2_{NCV,1} = 1.0$ m/s^2 for regimes of flight in which the target is not maneuvering. The process noise variance for Model 2 for tracking through target maneuvers is a function of range (r) selected as specified in [5], [6], [9] for a target with a maximum acceleration of A_{max} according to

$$\sigma_{NCA,2}(r) = \kappa_3(r)\frac{A_{max}}{\sqrt{3}} \tag{17}$$

where $\Gamma_D(r)$ is given by (12) and $\kappa_3(r)$ is given by (8). Mode switching probabilities are specified in (10) and (11).

Three cases, Cases 10-12, are considered in the study of IMM-CVCA design for radar tracking. For Case 10, $T = 1$ s, $\sigma_r = 5$ m, and $\sigma_a = \sigma_e = 1$ mrad, and Figure 11 gives the results for the Monte Carlo simulations that were conducted to illustrate the performance of the IMM-CVCA and NCV Kalman filter designed according (13) and (17), respectively. Note that the IMM-CVCA designed as described here gives similar results to the NCV Kalman filter during maneuvers and significantly smaller errors when the target is not maneuvering. The IMM-CVCA does not experience a peak error near that of the measurements at the beginning of the maneuver as in Case 7, because the accurate range measurements provide rapid maneuver response. For Case 11, $T = 0.25$ s, $\sigma_r = 5$ m, and $\sigma_a = \sigma_e = 1$ mrad, and Figure 12 gives the results for the IMM-CVCA and for NCV Kalman filter designed according (13) and (17), respectively. Note that the IMM-CVCA designed as described here gives similar errors to that of the NCV Kalman filter during maneuvers and significantly smaller errors when the target is not maneuvering. For $\sigma_a = \sigma_e = 2.5$ of Case 6, the $\Gamma_D(r = 130\text{km}) = 0.07$ and

Figure 1 indicates that at least 12 measurements are required to achieve a meaningful acceleration estimate. Hence, since maneuvers are less than 12 measurements, for Case 12, $T = 1$ s, $\sigma_r = 5$ m, and $\sigma_a = \sigma_e = 0.5$ mrad. Figure 13 gives the results to illustrate the performance of the IMM-CVCA and NCV Kalman filter designed according (13) and (17), respectively. In this case, $\sigma_a = \sigma_e = 0.5$ mrad are used instead $\sigma_a = \sigma_e = 2.5$ of Case 6 so that a meaningful acceleration can be estimated. Note that the IMM-CVCA designed as described here gives similar results to the NCV Kalman filter during maneuvers and significantly smaller errors when the target is not maneuvering. In this case, the IMM-CVCA does experience a single measurement spike at the beginning of two of the maneuvers. For these two maneuvers, the maneuver acceleration projects poorly in the range direction and this results in the spikes. The smaller angle errors in this case show the spikes as relatively larger errors. In Cases 10-12, the NCV Kalman filter clearly achieves MinMaxMSE in position as designed. The IMM-CVCA estimator designed as proposed here provides significantly better tracking when the target is not maneuvering and approximately achieves MinMaxMSE during maneuvers.

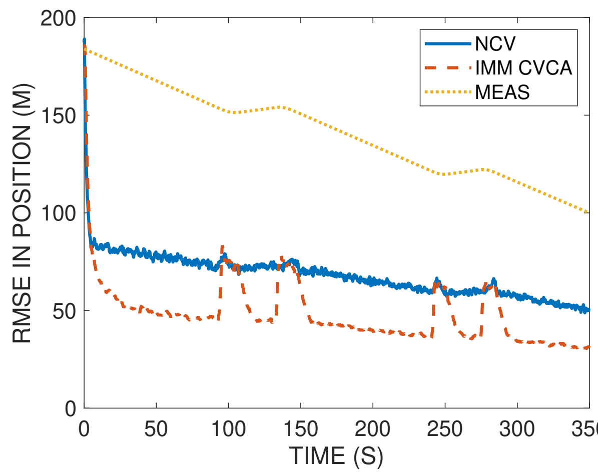

Fig. 12. RMSEs in position for IMM-CVCA and NCV Kalman filter radar tracking for Case 11 with $T = 0.25$ s, $\sigma_r = 5$ m, and $\sigma_a = \sigma_e = 1$ mrad and designed for MinMaxMSE via (13) and (17).

V. Concluding Remarks

Previously published design methods for the NCV Kalman filter [8] and NCA Kalman filter [6] were extended to develop design methods for the two-model IMM estimator with either two NCV models or NCV and NCA models. The design methods for the IMM-CVCV and IMM-CVCA are well-defined and automated in the selection of the process noise variance for the maneuver model. Further research is needed to improve the automated selection of the mode switching probability for changes in measurement rate. The question of when to use acceleration in a track filter is addressed in [9] and those results give insight into the selection between of the IMM-CVCV or IMM-CVCA for a specific application.

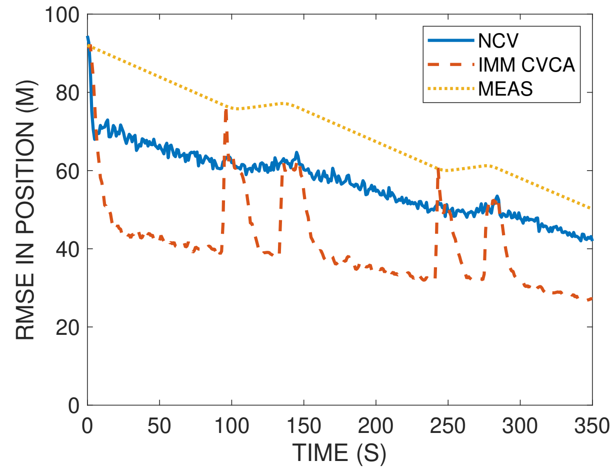

Fig. 13. RMSEs in position for IMM-CVCA and NCV Kalman filter for radar tracking for Case 12 with $T = 1.0$ s, $\sigma_r = 5$ m, and $\sigma_a = \sigma_e = 0.5$ mrad and designed for MinMaxMSE via (13) and (17).

References

[1] Y. Bar-Shalom, X. R. Li, and T. Kirubarajan, *Estimation with Applications to Tracking and Navigation*, John Wiley and Sons, Inc., New York, NY, 2001.
[2] Y. Bar-Shalom, P. K. Willett, and X. Tian, *Tracking and Data Fusion: A Handbook of Algorithms*, YBS Publications, Storrs, CT, 2011.
[3] S. Blackman and R. Popoli, *Design and Analysis of Modern Tracking Systems*, Artech House, Norwood, MA, 1999.
[4] W. D. Blair, "Design of Nearly Constant Velocity Filters for Tracking Maneuvering Targets," in *Proceedings of* 11^{th} *International Conference on Information Fusion,* Cologne, Germany, June 30-July 3, 2008.
[5] W. D. Blair, "Design of Nearly Constant Velocity Filters for Radar Tracking of Maneuvering Targets," in *Proceedings of 2012 IEEE Radar Conference,* Atlanta, Georgia, May 7-11, 2012.
[6] W. D. Blair, "Design of NCA Track Filters for Tracking of Maneuvering Targets," in *Proceedings of 2020 IEEE Radar Conference,* Florence, Italy, September 21-25, 2020.
[7] W. D. Blair and Y. Bar-Shalom, "Tracking Maneuvering Targets With Multiple Sensors: Does More Data Always Mean Better Estimates?" *IEEE Transactions on Aerospace and Electronic Systems*, January 1996, pp. 450-456.
[8] W. D. Blair and Y. Bar-Shalom, "MSE Design of Nearly Constant Velocity Kalman Filters for Tracking Targets With Deterministic Maneuvers" *IEEE Transactions on Aerospace and Electronic Systems*, March 2023.
[9] W. D. Blair and Y. Bar-Shalom, "On the NCA Versus NCV Models in Tracking Maneuvering Targets," in *Proceedings of 2023 IEEE Radar Conference,* San Antonio, Texas, May 2023.
[10] H. A. P. Blom and Y. Bar-Shalom, "The Interacting Multiple Model Algorithm for Systems with Markovian Switching Coefficients," *IEEE Transactions on Automatic Control*, August 1988, pp. 780-783.